# Direct Synthesis of Folded Symmetric Resonator Filters with Source-Load Coupling

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***Abstract*—A direct synthesis technique of coupled symmetric resonator filters with source-load coupling is presented. An examination of the implications of power conservation on the possible solutions to the synthesis problem is examined. It is shown that at least two different coupling matrices whose entries differ not only in signs but in magnitude, as well, can be extracted. The two matrices have the same coupling and routing scheme. Typical examples of synthesized filters are presented to document the validity of the technique.**

***Index Terms*—Bandpass filters, elliptic filters, resonator filters, synthesis.**

## I. Introduction

THE EXTENSIVE research on coupled resonator filters over the past few decades has been focused on topologies, where the source and the load are coupled to only one resonator each and not to each other [1]. It is well known that such a topology can produce at most $n-2$ finite transmission zeros out of $n$ resonators. The increasingly stringent requirements of modern communication systems in regards to flatness of group delay and stop band rejection can often be more expediently fulfilled by marshaling the effect of source-load coupling.

The addition of a direct signal path between the source and the load allows the generation of $n$ finite transmission zeros instead of $n-2$. Filters with source-load coupling have been presented by researchers in the past [2]–[4]. Despite the important effect of the source-load coupling on the response of the filter, the systematic synthesis of these components have not been exhaustively investigated. Although the source-load coupling can be accommodated within the synthesis technique presented in [5] and [6], they have not been applied to this type of filters to the author's knowledge.

Recently, a systematic synthesis technique of even-order symmetric filters with source-load coupling was presented by Montejo-Garai [4]. The technique is similar to the classic extraction technique introduced by Cameron [7] for filters with no source-load coupling with the exception that it starts with a parallel inverter instead of a unit element [4]. Examples of coupling matrices of orders 2 and 6 were given [4]. However, these coupling matrices exhibit some rather unexpected entries which makes their realization doubtful, especially for filters of orders higher than 2. It is shown in this paper that another solution, which allows the realization of filters with source-load coupling of any order, exists. We also propose to re–examine the implications of the unitarity of the scattering matrix of these filters on the possible steps of a successful synthesis.

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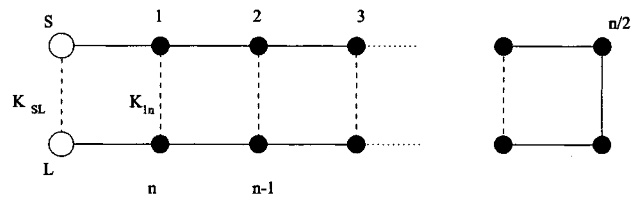


Fig. 1. Coupling and routing scheme of symmetric cross coupled resonators with source-load coupling.

## II. Theory

We focus attention on the symmetric network of coupled resonators shown in Fig. 1. The dark dots represent shunt capacitors in the lowpass prototype. The direct coupling (solid line) between adjacent resonators represent unit elements while the dashed lines, connecting resonator $i$ to $n-i+1$, are admittance inverters. For simplicity, we limit the discussion to the case of an even number of resonators. To generate $n$ finite transmission zeros, inverter $K_{sL}$ must be nonzero.

The specifications of the filter assume a given target function for the magnitude of the transmission coefficient $|S_{21}|$ [7] and [8]. Following the presentation in [7], the reflection and transmission coefficients $S_{11}$ and $S_{21}$ are extracted using the pole-zero approach. Since the specifications of the filter involve the magnitude of this coefficient only, its phase is arbitrary. We assume at this point that the numerators of both $S_{11}$ and $S_{21}$ are such that the coefficients of $s^n$ are *real* and that the coefficient of $s^n$ in the common denominator polynomial of both functions is equal to *unity*. However, for symmetric structures, the phases of $S_{11}$ and $S_{21}$ are required to satisfy the following condition:

$$\phi_{21} - \phi_{11} = \frac{(2n-1)\pi}{2}, \qquad n = 0, \pm 1, \ldots . \quad (1)$$

Here, $\phi_{21}$ and $\phi_{11}$ are the phases of $S_{21}$ and $S_{11}$, respectively. This condition can be straightforwardly derived from the unitarity of the scattering matrix (power conservation) using the property that $S_{11} = S_{22}$ for a symmetric structure. For the synthesis technique to succeed, this phase relationship must be fulfilled.

There are different ways of enforcing (1). The synthesis technique presented by Cameron [7] suggests starting the synthesis by extracting a unit element (90° phase shift). This operation has the effect of enforcing equation (1) when the starting functions

$S_{11}$ and $S_{21}$ have the same phase (or 180° out of phase). The extraction sequence is as follows.

1) Extract unit element.
2) Continue sequence as in [4].

On the other hand, the synthesis technique presented by Montejo-Garai starts by extracting the parallel inverter $K_{sL}$ [4] and presumably assumes that $S_{11}$ and $S_{21}$ already satisfy condition (1). The actual extraction of the elements of the network follow the formulas given by Cameron [7] and are not reproduced here.

A salient feature of networks with source-load coupling is that two different coupling matrices result depending on whether the right-hand side of (1) is $(\pi/2)$ or $-(\pi/2)$, as the examples show.

## III. Results

To illustrate the results which can be obtained from the technique presented here, we examine the two examples presented in [4].

The first example is a second-order filter with two transmission zeros at $s = \pm 7.5j$ and a minimum in-band return loss of 20 dB.

We assume that the coefficients of $s^2$ are all positive. Obviously, with this choice, these functions do not satisfy equation (1) and cannot represent a physically symmetric network. To restore the proper phase relationship, we start the synthesis by extracting a unit element from the input (there is a unit element on only one side, either input or output). The resulting coupling matrix, which is constructed from the capacitances and admittance inverters as described in [4], is

$$[M_1] = \begin{bmatrix} 0 & 26.8650 & 0 & 22.4569 \\ 26.8650 & 0 & 33.8025 & 0 \\ 0 & 33.8025 & 0 & 26.8650 \\ 22.4569 & 0 & 26.8650 & 0 \end{bmatrix}. \quad (2)$$

This coupling matrix is identical to the one given in [4] for the same filter.

The next possibility is to change the phase of either $S_{11}$ or $S_{21}$ by 180° and still start by extracting a unit element. The resulting coupling matrix is

$$[M_2] = \begin{bmatrix} 0 & 1.1963 & 0 & -0.0445 \\ 1.1963 & 0 & 1.6641 & 0 \\ 0 & 1.6641 & 0 & 1.1963 \\ -0.0445 & 0 & 1.1963 & 0 \end{bmatrix}. \quad (3)$$

If the synthesis is to be started by extracting the parallel inverter $K_{sL}$, it is necessary to reestablish the proper phase condition given in (1) before the synthesis commences. Carrying out the extraction steps, starting with the parallel inverter $K_{sL}$, it is found that the resulting coupling matrices are either $[M_1]$ or $[M_2]$ , as given by (2) and (3). The magnitudes of the reflection and transmission coefficients obtained from a direct analysis of the coupling matrices $[M_1]$ and $[M_2]$ are shown in Fig. 2. They agree with the original specifications of the filter for both matrices.

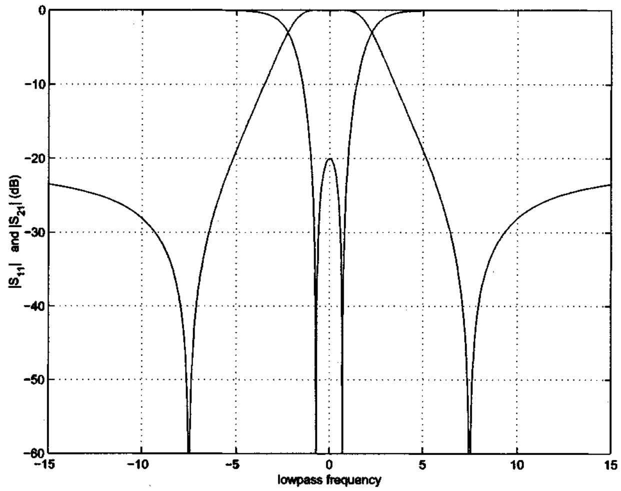


Fig. 2. $|S_{11}|$ and $|S_{21}|$ of second-order filter obtained from the coupling matrices $[M_1]$ and $[M_2]$.

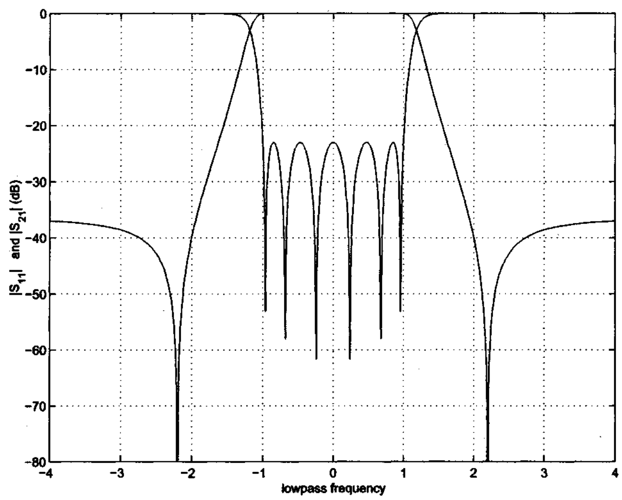


Fig. 3. $|S_{11}|$ and $|S_{21}|$ of sixth-order filter obtained from the coupling matrices $[M]$ (and $[M]$ given in [4]).

The second example is a sixth-order filter of minimum in-band return loss of 23 dB and transmission zeros located at $\pm 2.2j$ and $\pm 1.4 \pm 0.5j$. Following the same procedure, we obtain two coupling matrices, one of which is identical to that given in [4]. The other coupling matrix is given by (4), shown at the top of the next page.

The magnitudes of the reflection and transmission coefficients of this filter resulting from the analysis of matrix $[M]$ (and $[M']$ given in [4]) are shown in Fig. 3. The fulfillment of the original specifications shows the accuracy of the synthesis technique and the validity of the arguments presented here. An additional property of the two different coupling matrices for each of the two examples shown here is the fact that they

$$[M] = \begin{bmatrix} 0 & 1.0688 & 0 & 0 & 0 & 0 & 0 & -0.0075 \\ 1.0688 & 0 & 0.9094 & 0 & 0 & 0 & -0.0087 & 0 \\ 0 & 0.9094 & 0 & 0.6355 & 0 & 0.0964 & 0 & 0 \\ 0 & 0 & 0.6355 & 0 & 0.5310 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5310 & 0 & 0.6355 & 0 & 0 \\ 0 & 0 & 0.0964 & 0 & 0.6355 & 0 & 0.9094 & 0 \\ 0 & -0.0087 & 0 & 0 & 0 & 0.9094 & 0 & 1.0688 \\ -0.0075 & 0 & 0 & 0 & 0 & 0 & 1.0688 & 0 \end{bmatrix} \quad (4)$$

are not related by a similarity transformation since they have different eigenvalues, as a simple diagonalization shows.

## IV. Conclusion

It was shown that, depending on the choice of the phases of the transmission and reflection coefficients, two different coupling matrices which both have the same topology and give the same return and insert ion loss can be obtained for networks with source-load coupling. The coupling matrices are not related by a similarity transformation (rotation).

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